AdOc 4060/5060 Spring 2013 Chris Jenkins

Eddy viscosity

Surface Wind Stress (N/m²)



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Turbulence (video 1hr):

http://cosee.umaine.edu/programs/webinars/turbulence/?CFID=8452711&CFTOKEN=36780601

Part B

Surface wind stress

•Wind stress is the frictional force on the sea surface

$$\tau = \rho \operatorname{Cd} \operatorname{U}_{\operatorname{wind}}^2$$

- •Wind values 10m above surface are used as standard (U_{10})
- •Surface water currents are \approx 3 to 10% of U_{wind}

Approximately: $c_D = 1.5 \times 10^{-3}$

(dimensionless)



Photo of the sea state under Category 4 Hurricane Isabel taken from 400 feet above the surface. Note that the aircraft was not in or near the eyewall at this time or altitude. (Will Drennan, RSMAS)

http://www.huffingtonpost.com/2012/11/02/what-happens-underwater-during-a-hurricane_n_2066084.html

Eddy Viscosity

Viscosity:



Basic physics

$$au = F/A$$

 $au = -
ho v rac{dV}{dZ}$

Viscosity: resistance to shearing flows, where adjacent layers move parallel to each other with different speeds



• Turbulence tends to flux momentum down the gradient of the mean flow in an analogous fashion to the viscous transfer of momentum.

• Thus the turbulent flux of momentum can be parameterized in terms of a *down-gradient flux* with an *eddy viscosity*

$$\begin{split} \nu_{ed} \sim 10^{-3} \; \frac{\mathrm{m}^2}{\mathrm{s}} \; \gg \; \nu = 10^{-6} \; \frac{\mathrm{m}^2}{\mathrm{s}} \\ \end{split} \\ \text{Typical eddy viscosity in} \\ \text{the upper ocean} \; & \text{Kinematic (molecular)} \\ \text{viscosity of water} \end{split}$$



Schlieren Imaging of Turbulence: http://rses.anu.edu.au/research/ep/gfd/projects.php

Mixing transfers Momentum

In the ocean, measurable diffusion occurs by turbulent processes (these eddies and swirls described by eddy coefficients). Turbulent eddies have scales that vary from tens of mm to tens of km and thus the range in values is $A_V = 1-10$ g/cm s

 $A_{\rm H} = 10^3 - 10^7$ g/cm s where the subscripts H and V denote horizontal and vertical.



is the gradient in z-directed momentum transfer derived from x-directed currents (U)

Ax, Ay and Az are the eddy viscosities

"A coefficient used to achieve *closure* in the *Reynolds equations* for turbulent flow. The assumption is made that the *Reynolds stresses* are related to the velocity gradients of the flow by a viscosity analogous to the molecular viscosity, i.e. a turbulent or *eddy viscosity*. The utility of the analogy is strained by the fact that while the molecular viscosity is a property of the fluid, the eddy viscosity is a property of the flow. As such the specification of the eddy viscosity has more than a little of the air of the ad hoc about it since it is usually found via a trial-and-error procedure wherein it is varied until a numerically simulated flow reasonably replicates a known flow. The value thus obtained diagnostically is then used for prognostic simulations, a procedure that is questionable due to the abovementioned fact of the eddy viscosity being a property of the flow rather than the fluid. That is, if the flow is remarkably different, then the eddy viscosity may also be remarkably different."

http://stommel.tamu.edu/~baum/paleo/ocean/node9.html

"...the ability of eddy viscosity models to drain energy from large scales, thus simulating the dissipative nature of turbulence..."

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19960022300_1996039613.pdf

The Boussinesq approximation

• Because the ocean is stably stratified, vertical turbulence is greatly depressed compared to horizontal turbulence

• There are flows in which the temperature varies little, and therefore the density varies little, yet in which the buoyancy drives the motion. Thus the variation in density is neglected everywhere except in the buoyancy term.

Useful for simplifying the equations of fluid motion, the Navier Stokes Equations

Ekman Transport scalings use Eddy Viscosity

Ekman deduced that (in a homogeneous infinite ocean) the speed of the surface current u_0 is:

$$u_0 = \frac{\tau}{\rho \sqrt{A_z f}}$$

$$\tau = \text{wind stress on ocean surface}$$

$$f = 2\Omega \sin\phi,$$

$$A_z = \text{eddy viscosity for vertical mixing}$$

$$\rho = \text{density of the seawater}$$

$$D = \text{reversal depth}$$

speed of the net current \overline{U} is:

 $Q = \frac{\tau_{wind}}{\rho f}$

Q, transport

http://marine.rutgers.edu/dmcs/ms501/2004/Notes/Wilkin20041025.htm

100 meters

Doppler radar current measurement



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Direct Observations of Fronts



08-12-2003 19:00:00 GMT



http://www.cds.caltech.edu/~shawn/LCS-tutorial/oceancurrents.html

Part B

Turbulent and laminar flow

- 1. Laminar flow -smooth, unidirectional flow of low velocity Laminar flow is rare
- 2. Turbulent flow disordered, multidirectional flow

Turbulent Mixing occurs perpendicular to current

- a. Transition from laminar to turbulent flow occurs at the interface of two miscible fluids moving in opposing directions
- b. Critical velocity is a function of viscosities and densities



Reynolds Number

$\operatorname{Re} = \frac{\rho V D}{\mu}$:

where:

- is the mean fluid velocity in (<u>SI units</u>: m/s)
- •*D* is the diameter (m)
- • μ is the <u>dynamic viscosity</u> of the <u>fluid</u> (Pa·s or N·s/m²)
- •v is the kinematic viscosity (v = μ / ρ) (m²/s)
- • ρ is the <u>density</u> of the fluid (kg/m³)

http://en.wikipedia.org/wiki/Reynolds_number

Transition Reynolds number

In <u>boundary layer</u> flow over a flat plate, experiments can confirm that, after a certain length of flow, a laminar boundary layer will become unstable and become turbulent. This instability occurs across different scales and with different fluids, usually when Re_x=5x10⁵, where *x* is the distance from the leading edge of the flat plate, and the flow velocity is the 'free stream' velocity of the fluid outside the boundary layer.

Reynolds number sets the smallest scales of turbulent motion

In a turbulent flow, there is a range of scales of the time-varying fluid motion. The size of the largest scales of fluid motion (sometime called eddies) are set by the overall geometry of the flow. For instance, in an industrial smoke stack, the largest scales of fluid motion are as big as the diameter of the stack itself. The size of the smallest scales is set by the Reynolds number. As the Reynolds number increases, smaller and smaller scales of the flow are visible. In a smoke stack, the smoke may appear to have many very small velocity perturbations or eddies, in addition to large bulky eddies. In this sense, the Reynolds number is an indicator of the range of scales in the flow. The higher the Reynolds number, the greater the range of scales. The largest eddies will always be the same size; the smallest eddies are determined by the Reynolds number.

Turbulence Dissipation



Schematic representation showing the form of the energy spectrum of turbulent velocity cascade, where E(k) is the spectral density and k is a wave number ... The kinetic energy generated at large-scale L cascades through the inertial subrange, i.e. a hierarchy of eddies of decreasing size to the viscous Kolmogorov scale I_k , where it is dissipated into heat.

http://www.sciencedirect.com/science/article/pii/S0967063700000893

$$l_k = \eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$
 Kolmogorov scale

 ϵ Turbulence dissipation rate (J kg⁻¹ s⁻¹)

... at very high Reynolds number the statistics of scales in the range $\eta \ll r \ll L$ are universally and uniquely determined by the scale *r* and the rate of energy dissipation http://en.wikipedia.org/wiki/Turbulence

the energy spectrum function according with the third Kolmogorov's hypothesis is $E(k) = C \varepsilon^{2/3} k^{-5/4}$, where C would be a universal constant.

Here, k is the spatial wave number, $k=2\pi/L$; Coefficient C=~2.0

Eddy Viscosity & Turbulence

 $A_l \approx \left(\varepsilon * l\right)^{-1/3}$



Plankton encounter & turbulence: http://vimeo.com/ 59342465





http://www.ucmp.berkeley.edu/arthropoda/crustacea/maxillopoda/copepodalh.html

http://www.sciencedirect.com/science/article/pii/S138511010200120X

Langmuir Circulation



