quick study

The surprising motion of ski moguls

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Regularly spaced bumps that arise on ski slopes defy intuition by migrating uphill, even though skiers and snow move downhill.

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Ski moguls form on virtually all ski runs that are not mechanically flattened with grooming equipment. Those amazingly ordered structures are not planned or constructed; they organize spontaneously as a consequence of skiers turning and moving snow. Although phenomena that arise from self-organization are common, the moguls' high visibility, ubiquity, and regularity make them a particularly surprising and impressive consequence of such seemingly random actions as ski turns. Ostensibly, skiers can turn when and where they please. Moreover, a skier's turning radius depends on a variety of factors, including ski length and shape, snow conditions, skier ability, and the details of the skier's knees and legs, which act as damped springs with a characteristic frequency. Nevertheless, the independent acts of many skiers form rows of moguls that not only space themselves in a regular checkerboard pattern (see panel a of the figure) but also migrate over time. And, although skiers invariably push snow down the mountain, the ski moguls move uphill.

Kinematic waves

Skiers navigating a slope with large bumps cannot, in fact, turn where they please. Those who turn on the uphill side of a mogul will experience severe knee compression, which can lead to painful ligament injuries. To control speed, skiers turn and scrape the snow on the downhill side of the moguls they encounter. In so doing, they push snow down the mountain and pile it onto the uphill side of the following mogul. As a consequence, each mogul loses material on its downhill side but gains new material on its uphill side, as shown in panel b of the figure. The net effect is that the moguls migrate uphill. Time-lapse videos, available with the online version of this Quick Study, show the migration speed to be roughly 0.08 m/day.

Moguls are a type of kinematic wave, an entity rather different from the more commonly studied dynamic wave. Dynamic waves—ocean waves, for example—are a consequence of Newton's second law and do not reflect net transport of material; an ocean wave will pass underneath a stationary swimmer. In contrast, kinematic waves are traveling quantities or shapes—say, moguls on a ski slope—governed by mass or other conservation principles. Consider highway traffic, for example. If a car taps its brakes, then the cars behind bunch up and the density of cars increases. That bunching travels backward through the traffic, even though the cars continue to move forward, and so the bunched cars, like moguls, are said to be backward propagating. The number of cars is conserved. As the kinematic wave passes through the conserved material, the density of the cars changes as they bunch up to avoid a collision. On the ski slope, the amount of snow is conserved. As moguls pass through the snow, the thickness of snow changes as skiers continuously sculpt the landscape.

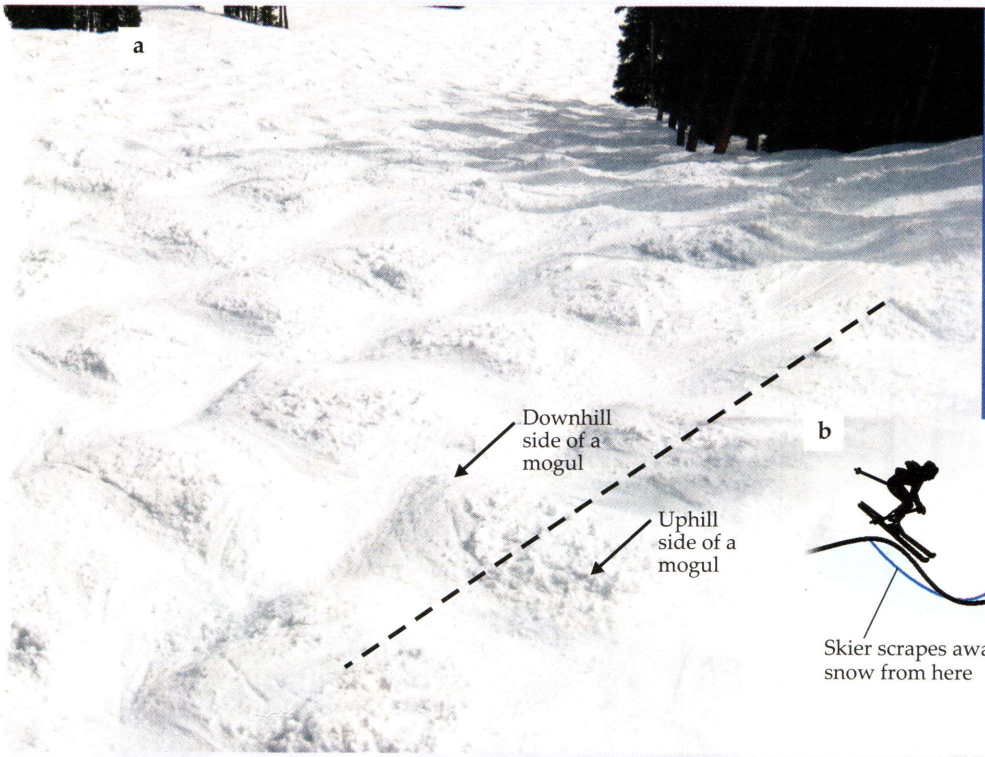
Making a mogul

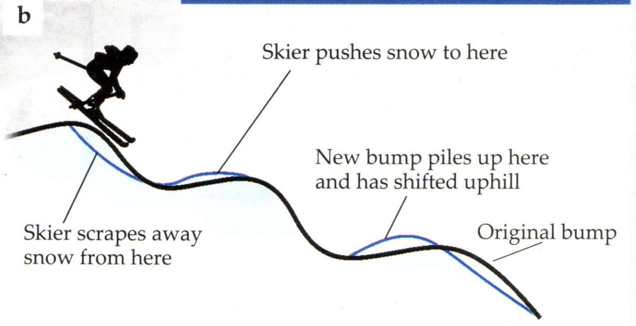
Skiers ignore bumps whose height is less than a critical value h. While skiing down a previously unskied area that is covered with snow to a uniform depth, they will make S-shaped turns that cross over small piles of snow deposited by other skiers. The wavelength λ, turning radius $r = \lambda/4$, and phase ϕ of each skier's path are effectively random and independent of the corresponding parameters for other skiers. But once a pile has reached the critical height, skiers' knees will alert them that they need to turn on the downhill side of the proto-mogul.

In a single turn of diameter $2r$, the height of the snow surface will decrease where the skier erodes snow and will increase where the skier deposits snow. That cyclic erosion and deposition can be represented as a sine wave, square wave, or any other wave with wavelength $2r$, half that of the skier's path. The waveform's amplitude a and morphology depend on the skier's ability, the length of the skis, and so forth. Even snow conditions play a role in determining a, but a typical value is about 1 cm or so for hard-packed snow. A specific representation for the erosion–deposition wave W at position x created by a skier n may be given by the sinusoidal form $W_n(x) = a \sin(2\pi x/2r_n + \phi_n)$, with positive W corresponding to deposition. The subscripts are reminders that the erosion–deposition wave and its defining parameters can vary for different skiers; for simplicity, we assume that the amplitude is the same for all skiers.

Ironically, the randomness of the skiers' turns guarantees the orderly development of moguls. Given enough skiers and random phase shifts and wavelengths, the law of large numbers implies that the average value of $W_n(x)$ will tend to zero for all x. However, at any specific time and place on the ski slope, the sum of erosion and deposition is a random accumulation of values between $-a$ and a. Therefore, at some unspecified but definite time, the sum (as opposed to the average) will deviate far enough from zero that a bump will reach the critical height. From then on, skiers can no longer ignore it.

If, for example, the erosion and deposition are approximated as a square wave rather than a sine wave, then the sum





Form and formation of moguls. **(a)** Moguls on Riflesight Notch at Colorado's Winter Park Resort display a characteristic checkerboard pattern. Downhill is toward the top of the photograph; the mogul field shown here is about 100 m long. The uphill side of each mogul has loose material deposited by skiers; the downhill side is worn smooth by skier erosion. Moguls are separated by approximately 5.7 m. **(b)** Moguls migrate uphill because skiers erode the downhill side of each bump and deposit snow on the uphill side of the following bump.

represents a random walk about zero. In that case the expected deviation from zero, $aN^{1/2}$, varies as the square root of the number of skiers N. Bumps will thus reach the critical height when $N = (h/a)^2$. If you observe ski slopes, you'll see that moguls form quickly; some hundreds of skiers, and frequently fewer, are enough to form them. If we estimate that $N \approx 100$ skiers are enough to generate bumps of the critical size, then $h/a \approx 10$. The critical height, then, is about 10 times that of the erosion–deposition amplitude, roughly 0.1 m for hard-packed snow. Once the proto-moguls have formed, skiers tend to turn on the downhill side. That preference slowly realigns each bump's position until moguls have organized into the classic checkerboard pattern.

Numerical models can elucidate the cumulative effect of skiers moving over snow. One model treats skiers as a fluid moving over sediments; the moguls are analogous to the ripples that form when a river flows over sand. Other models may be borrowed from studies of sand dunes, washboard patterns created by cars on dirt roads, or other structures that make up backward-propagating kinematic waves. A dimensional analysis, however, can answer many questions while sidestepping more complicated approaches. The mogul-to-mogul distance l should depend on a typical skier's turning radius and speed v, the gravitational acceleration g, and the angle of the ski slope θ. The slope is dimensionless, and the turning radius, velocity, and gravitational acceleration are not independent quantities. Thus two pertinent dimensional relationships can be deduced, $l \propto r$ and $l \propto v^2/g$. The first says, not surprisingly, that the separation of moguls is proportional to a typical turning radius. The second is similar to an expression that arises when considering washboard stripes on dirt roads. It explains why moguls are usually longer at the end of a series of bumps—skiers tend to move faster as they approach the groomed terrain at the bottom of a ski run, where a fall would be less dangerous.

Earn that beer

Moguls migrate uphill only because skiers expend energy when eroding and transporting snow. How much energy does it take for skiers to move a mogul? Measurements show that recreational skiers expend about 20–25 kilocalories per minute. Skiers spend roughly 20 seconds of actual time skiing (as opposed to huffing and puffing) when traversing a 100-m mogul field such as the one shown in the figure, located at Riflesight Notch in Colorado. That means a skier expends 8 kcal in a run. About 10 skiers go down Riflesight Notch each hour, and the mogul field is open 50 hours per week for five months. So there are about 10 000 runs down Riflesight Notch per season and 80 000 kcal expended. The field has about 200 moguls; that comes to 400 kcal per mogul each season. Since moguls move about 10 m uphill over the course of a season, each one requires 40 kcal/m to move uphill. More prosaically, skiers expend half a light beer for every meter of uphill mogul movement—a lot of work for not a lot of refreshment.

Turning the above analysis around, observations of mogul migration yield the amount of time skiers spend actually skiing and, given a count of the number of individual skiers, the amount of time and energy a typical skier spends exercising. In our increasingly obese society, such indirect measures of physical-activity energy expenditure have become extraordinarily valuable for public health officials.

Additional resources

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▶ N. Taberlet, S. W. Morris, J. N. McElwaine, "Washboard Road: The Dynamics of Granular Ripples Formed by Rolling Wheels," *Phys. Rev. Lett.* **99**, 068003 (2007).