A Seasonal Zonal Energy Balance Climate Model With an Interactive Lower Layer

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The mean annual energy balance climate model of Gal-Chen and Schneider (1976) is expanded to a more general model which includes an interactive lower layer. The two-layer model is used to simulate the seasonal cycle through the use of seasonally varying insolation. Rather than greatly modifying the zonal model parameterizations, we choose to determine the extent to which the use of the present parameterizations in a completely zonally averaged model can reasonably simulate the seasonal cycle of surface air temperature and meridional heat transport. It is found that the model-derived cycle of surface air temperature lags the observations by 1-2 months, but the amplitude of the seasonal cycle can be well simulated by using reasonable annual mean values of the seasonally effective zonal thermal inertia. The seasonal variations in the meridional transport of energy by the atmosphere agree qualitatively in midlatitudes with the data of Oort (1971), but they suffer from large errors in the tropics. Seasonal simulation indicates that the diffusive atmospheric energy transport parameterization based on annual data is inappropriate in this region. Comparisons of annual and seasonal models show that there is little difference in temperature sensitivity for solar constant changes. Unlike previous low-resolution climate models the seasonally effective thermal inertia is also allowed to vary with time to simulate the seasonal variation of the oceanic mixed layer depth. This modification requires the addition of a second lower vertical layer in the model, the temperature of which is predicted explicitly. Such seasonal thermal inertia variations have little effect on the model's equilibrium response to solar constant changes. Experiments employing step function and exponential solar constant increases show the time-dependent response of global surface temperature to lag the solar constant perturbation by from a few years to a few decades, depending on the assumptions of seasonal thermal inertia variation and lower layer thickness. The uncertainty in the range of global temperature lag time implies that modeling the time-dependent temperature response to a CO₂ perturbation will require refined treatment of the coupling between upper and lower oceanic heat reservoirs. The most important general conclusion from these experiments is that realistic values of seasonally effective thermal inertia (i.e., primarily the oceanic mixed layer depth) are needed for the realistic simulation of the seasonal cycle of temperature. Use of realistic seasonal thermal inertia implies that climate sensitivity experiments with seasonal models (including global circulation models) will require decades of model simulation time to approach a reasonable climatic equilibrium.

I. INTRODUCTION

In recent years there has been considerable interest in the highly parameterized approach to global climate modeling afforded by energy balance climate models. The basic premise of such modeling is that the large-scale zonally averaged features of earth's climate can be simulated by parameterizations based solely on empirical functions of the surface temperature. Although much research remains to be done before the extent to which this premise is true can be proven, we feel that these low-resolution, highly parameterized models have an important role to play in our developing knowledge of the workings of climate. In a review, Schneider and Dickinson [1974] discuss the presently evolving hierarchy of numerical models for simulating the global climate. An inherent limitation of the largest, most physically comprehensive models, the general circulation models, is that they require enormous amounts of computational time. For this reason it can be advantageous to use the highly parameterized models to investigate the sensitivity of the global climate to changes in the energy flows both within and external to the land-ocean-atmosphere-cryosphere system. Moreover, it is verification against real data, not merely the physical comprehensiveness of a model, that should give us confidence in its performance. We intend here to develop a model based on annual data but then to verify aspects of its performance with seasonal simulations. The extent to which the seasonal cycle of insolation is able to produce variations in the model climate close to those in the real climate on comparable time and space scales will help to tell us how the model performs. Adding physical processes to a model is no guarantee that these processes have been parameterized correctly or that other processes not included would negate the influence of those which are included. Thus our intention is to see how much of the zonally averaged seasonal variation in surface temperature can be explained by a simple energy balance model. This comparison with observed data is a major object of this study.

Sellers [1969] and Budyko [1969] introduced the two archetypal semiempirical energy balance models that have formed the basis of most subsequent work. As a result of the introduction of surface albedo-temperature feedback (i.e., more snow and ice is associated with colder temperatures-and the converse), these mean annual models show a large temperature sensitivity to changes in the solar constant, particularly in the polar regions. This feedback amplified the change in global surface temperature for a 1% decrease in the solar constant over an experiment with no albedo-temperature feedback from approximately -1.5°K to -4.0°K, suggesting that this mechanism is important for climatic change. This characteristic of the Sellers and Budyko models led Schneider and Gal-Chen [1973] to formulate a time-dependent numerical model incorporating features from both the previous models. They found that their numerical model was transitive to a large range of initial temperature perturbations (i.e., the equilibrium solution was independent of the initial condition). However, they also found two stable intransitivie steady states:

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the present-day climate and an ice-covered earth. Further work by Gal-Chen and Schneider [1976] compared different parameterizations for the planetary albedo and the meridional transport of heat. Ghil [1976], Held and Suarez [1974], and Chýlek and Coakley [1975] have analyzed analytically both the Sellers- and Budyko-type annual mean models and concluded also that they possess only two mathematically stable temperature solutions for the current value of the solar constant, namely, the present climate and the ice-covered earth. North [1975a, b] has examined analytic diffusive meridional heat transport models and has also obtained two stable solutions for the present solar constant. Drazin and Griffel [1977] have shown that a North-type analytic climate model can possess asymmetric solutions (e.g., an ice cap at only one pole), one of which was shown to be stable for the present solar constant.

The large global temperature sensitivity of the Sellers- and Budyko-type models, which enhanced their original interest, has been lessened recently as various parameters have been shown to be uncertain or in need of modification. Warren and Schneider [1979] demonstrate that the uncertainty in existing radiation parameterizations can result in the existence of a non-ice-covered earth solution for solar constant decreases as large as 10% or more. Using a larger temperature sensitivity in their infrared parameterization, Oerlemans and van den Dool [1978] decreased the global temperature sensitivity of their model, a possibility earlier demonstrated by Schneider and Mass [1975]. Models including a presumed solar zenith angle dependence of cloud albedo (Ohring and Adler [1978] and Coakley [1979]: for a discussion of the zonal albedo of clouds. see Cess [1976]) also show a reduced temperature sensitivity, since albedo-temperature feedback need not be as strong as it was in earlier models.

One of the most difficult problems in climate modeling at any stage in the model hierarchy is that of verifying a model's ability to simulate climate changes accurately. Even the most complex global circulation models (GCM's) are linked to the presently observed climate by empirical constants or fixed cloudiness, have questionable parameterizations of some subgrid scale processes, or incorporate simplifications such as noninteractive albedo or sea surface temperature. Highly parameterized, semiempirical energy balance models are also explicitly linked to the current climate, but the linkages differ in scale from the GCM's. Although the annual mean models mentioned above show considerable ability to reproduce the observed annual mean distribution of surface temperature. this is not unexpected in view of the fact that they have been at least partially 'tuned' to the present climate. The lack of independent verification of a model's sensitivity to external forcings raises questions about the validity of the results of climatic change sensitivity experiments. One possible method of quasi-independent verification is to use a model that is linked only to the current annual conditions (i.e., uses only mean annual data to derive its empirical constants) to simulate seasonal changes of meteorological variables such as surface temperature. A reason for performing seasonal experiments is that a model which responds correctly to the seasonal cycle of insolation is more likely to respond correctly than an annual model to other external perturbations (like a CO₂ doubling).

Wetherald and Manabe [1972] examined the response of an ocean-atmosphere GCM to the seasonal variation of solar radiation and found a significant high-latitude warming as compared with an annual model. Sellers [1973] produced a quasi-zonal, seasonal semiempirical model by dividing latitude zones into land and ocean fractions and computing a temper-

ature for each. His model simulated many of the essential features of the global climate, including the seasonal cycle of surface temperature and meridional heat transport. *Robock* [1978] has used a model like *Sellers* [1973] to simulate climate change forced by random fluctuations in meridional atmospheric heat transport, volcanic dust variations, sunspot-related solar constant changes, and anthropogenic forcing (CO₂, aerosols, and heat). Our present work presents the results of extending the *Gal-Chen and Schneider* [1976] model to simulate the cycle of the seasons. The model will remain purely zonal (land and sea are not distinguished) and will employ essentially the same parameterizations as the previous mean annual version. We will, however, include an additional lower layer in the model in order to perform experiments using time-varying thermal inertia.

Aside from the now standard experiments involving sensitivity to solar constant changes, there are several experiments that can be performed only with a seasonal model: (1) We would like to determine if the annual mean cliamte of a seasonal model differs from that of an annual mean model. (2) Since we know there is a seasonal variation in both the planetary albedo and the insolation, we suspect there may be a significant interaction of these two variables, resulting in a residual mean annual temperature change which could be simulated only by a seasonal model. Therefore we have run the model using three different specifications for albedo and have compared the results. (3) In previous zonal energy balance modeling studies, the depth of the oceanic mixed layer, which is proportional to the heat capacity of the planet for seasonal time scales, was assumed to be constant in time. We have attempted to simulate the effects of the seasonal variation of the mixed layer depth by specifying a time-varying, seasonally effective heat capacity or seasonal thermal inertia. (By 'seasonally effective' and 'seasonal' heat capacity or thermal inertia we mean the thermal inertia which is appropriate for time scales ranging up to a few years. Of course, the vast heat capacity of the entire world ocean makes the very long term climatological thermal inertia of the earth much larger than that which determines the amplitude of the seasonal cycle of temperature.) (4) Semiempirical models usually assume that meridional heat fluxes can be expressed by a nonlinear diffusion law (Stone type), a linear diffusion law (Sellers type), or a linear 'Newtonian cooling' heating law (Budyko type). While the diffusive transport parameterization may be expected to work in the mid-latitudes where heat transport by large-scale eddies predominates, its use in the tropics has been largely a matter of convenience (particularly for analytic models) rather than a belief of its validity. The linear heating law is very general and does not break down the energy transport into the individual latent, sensible/potential, and oceanic terms. Lindzen and Farrell [1977] strongly suggest that the diffusive- and linear-type parameterizations may not be applicable to the tropics. We will confirm this by comparing the seasonal models' meridional heat flux with the seasonal observations of Oort [1971].

2. THE SEASONAL MODEL

Unlike the model of *Schneider and Gal-Chen* [1973], there are two basic governing equations in the general seasonal model developed here. They are both time dependent, zonally averaged energy balance equations, each vertically integrated over its respective domain. The top-layer temperature refers to a bulk land, ocean-mixed layer, atmosphere temperature, which is assumed to be proportional to the zonally averaged



Fig. 1. A schematic diagram of the two-layer model. The solid arrows represent the explicit diffusion of energy between latitude zones. The dashed arrows represent the transfer of heat between layers as a consequence of the time variation of R. The sum of the thermal inertias of the two layers is constant in time but does vary with latitude ϕ . The crosshatched areas are insulated 'walls' through which no heat transfer occurs.

surface air temperature. The bottom layer provides a thermal source or sink to account for energy lost or gained by the timevarying upper layer over the course of an annual cycle. The total heat capacity of upper and lower layers is time invariant, although the capacity of each layer varies with time.

The following energy balance equation was used by Schneider and Gal-Chen [1973]:

$$R(\phi)(\partial T/\partial t) = Q_n - \operatorname{div} F \tag{1}$$

where

 $R(\phi)$ thermal inertia, J m⁻² °K⁻¹;

- $T(\phi, t)$ surface temperature;
 - t time;
 - ϕ latitude;
- $Q_n(\phi, t, T)$ net radiation at the top of the atmosphere, W m⁻²;
- $F(\phi, t, T)$ northward transport of heat by oceans and atmosphere, W m⁻¹.

This equation can be shown to occur as a special case where R is not a function of time in a more general two-level model.

To derive the more general model, a schematic of which is shown in Figure 1, we consider the following energy balance equations at each latitude and assume that any time variation in R is produced by the seasonal variation in the oceanic mixed layer depth. Now the zonal equivalent heat capacity is $R(\phi, t)$, and

$$(\partial/\partial t)(RT) = Q_n - \operatorname{div} F + T(\partial R/\partial t) \qquad \partial R/\partial t < 0$$
 (2)

Thus when the equivalent mixed layer becomes thinner $(\partial R / \partial t < 0)$, the last term on the right shows that energy is lost simply because mass is lost at the temperature T. This energy is conserved by assuming that the mass lost by the top layer is simultaneously gained by a bottom layer which is at temperature T_0 . The heat balance equation for the bottom layer is then

$$(\partial/\partial t)(R_0T_0) = -T(\partial R/\partial t) \qquad \partial R/\partial t < 0 \qquad (3)$$

where $R_0(\phi, t)$ is the thermal inertia of the bottom layer. We specify that $\partial R/\partial t = -\partial R_0/\partial t$, or

$$R_t(\phi) = R(\phi, t) + R_0(\phi, t) = \text{constant in time}$$
(4)

where R_t is the total thermai inertia of both layers at any latitude. This specification conserves mass and, therefore, energy in the two-level system. From (3) we see that the temperature of the bottom layer is only influenced by that of the top layer; no radiative or meridional heat fluxes occur into or out of the bottom layer.

Similarly, when the top layer deepens, we have

$$(\partial/\partial t)(RT) = Q_n - \operatorname{div} F + T_0(\partial R/\partial t) \qquad \partial R/\partial t > 0 \quad (5)$$

$$(\partial/\partial t)(R_0T_0) = -T_0(\partial R/\partial t) \qquad \partial R/\partial t > 0 \qquad (6)$$

Equations (2), (3), (5), and (6) can be rewritten as prognostic equations for T and T_0 :

$$R(\partial T/\partial t) = Q_n - \operatorname{div} F \qquad \partial R/\partial t < 0 \tag{7}$$

$$R_0(\partial T_0/\partial t) = (T_0 - T)(\partial R/\partial t) \qquad \partial R/\partial t < 0 \qquad (8)$$

$$R(\partial T/\partial t) = Q_n - \operatorname{div} F + (T_0 - T)(\partial R/\partial t)$$

$$\partial R/\partial t > 0$$
 (9)

$$R_0(\partial T_0/\partial t) = 0 \qquad \partial R/\partial t > 0 \qquad (10)$$

T is only affected by T_0 when the top layer is deepening, at which time T_0 is constant because lower-layer water is being mixed into the upper layer, not vice versa. T_0 can change, however, when $\partial R/\partial t < 0$. If $\partial R/\partial t = 0$, then the bottom layer does not interact with the top and the equation governing T reduces to (1). An alternative to computing T_0 directly is to specify an initial distribution of $T_0(\phi)$ and update it annually by the amount that the surface temperature changes. The shortcomings of this method are described in Appendix B.

A problem arises in physically interpreting temperatures in the zonally averaged two-layer model because a model zone consists of a mixture of land, sea, and atmosphere, requiring that R be some effective zonal thermal inertia determined by the relative fractions of these components-a determination that is subject to considerable ambiguity, as we shall see later. Furthermore, the seasonal variation of R should be primarily due to the mixed layer depth variation in the oceanic fraction of a zone. However, T is a composite ocean/land surface air temperature, not the oceanic mixed layer temperature, and T_0 is the temperature that results from the energy balance constraint. T_0 need not be a good approximation to a permanent thermocline temperature, since it represents both land and sea components of a zonal average. These ambiguities in physical interpretation of T_0 can only be rectified by using a model which treats land and ocean (or even atmosphere) temperatures separately. We realize that higher-resolution models can-and should-be built, but first we want to understand the capabilities and limitations of the present zonal model through tests against independent data.

The net radiation at the top of the atmosphere is simply

$$Q_n = Q(1 - \alpha) - I \tag{11}$$

where $Q(\phi, t)$ is the extraterrestrial insolation, $\alpha(\phi, t, T)$ is the planetary albedo, and I(T) is the outgoing infrared radiation. The term div F is the divergence of energy transported meridionally by the atmosphere and oceans. F is the sum of three terms: the energy transported as the latent heat of water vapor (F_l) , the transports of atmospheric sensible and potential energy (F_a) , and the transport of energy by ocean currents (F_0) . Thus $F = F_l + F_a + F_0$, and

div
$$F = (\cos \phi)^{-1} (\partial / \partial y) (F \cos \phi)$$
 (12)

TABLE 1.Zonal Values of Observed Mean Annual Temperature \overline{T} , Observed Planetary Albedo α , Insolation \overline{Q} , and the Consistency
Factor c

Latitude	₹, °K	ā	\overline{Q} , W m ⁻²	С
80°N	257.2	0.567	177.3	1.050
70°N	263.5	0.501	195.6	1.070
60°N	272.2	0.435	235.0	1.080
50°N	278.0	0.390	282.9	1.060
40°N	285.8	0.341	327.2	1.026
30°N	293.0	0.295	364.3	0.984
20°N	298.0	0.265	392.1	0.979
10°N	299.2	0.255	409.4	0.999
0°	299.8	0.252	415.5	1.000
10°S	298.9	0.240	409.9	0.950
20°S	296.4	0.245	393.1	0.927
30°S	292.4	0.277	365.8	0.955
40°S	286.4	0.330	329.1	0.998
50°S	279.3	0.395	285.2	1.037
60°S	272.0	0.471	237.5	1.053
70°S	255.0	0.555	198.5	1.023
80°S	233.0	0.608	180.1	0.943

where $y = a\phi$, increasing northward, and a is the radius of the earth.

In our model the planetary albedo α can be held constant at its observed insolation-weighted mean annual value (denoted by MA) or allowed to vary with time in a predetermined manner so as to reproduce the observed seasonal changes; i.e., albedo is imposed seasonally (IS). Alternatively, albedo can be made a function of the surface temperature so that albedotemperature feedback (ATF) occurs. The formulation used for ATF is that of Sellers [1969]. The albedo-temperature feedback constant c_{f} is taken to be $-0.009^{\circ}K^{-1}$, although more recent work by Lian and Cess [1977] suggests a global value of about $-0.004^{\circ}K^{-1}$ to be more consistent with limited observations. T_{f} , the cutoff temperature above which albedo is no longer a temperature-dependent function, is taken to be 283.15°K (10°C). Although the Sellers [1969] albedo parameterization has not been verified, it is used here because it has been used extensively in the past. Since we want to compare the seasonal model's results to those of a mean annual model similar to a previous model [Gal-Chen and Schneider, 1976], we will not change the temperature-dependent albedo parameterization.

The outgoing infrared radiation is given by

$$I = a_0 c(\phi) + bT \tag{13}$$

which is a linearization of the empirical formula of Sellers [1969]. The constants a_0 and b are -229.8 W m⁻² and 1.62 W m⁻² °K⁻¹, respectively. The term $c(\phi)$ is a consistency factor of order unity which is described more fully by Schneider and Gal-Chen [1973]. Unlike the earlier work, $c(\phi)$ here multiplies a_0 , not the infrared sensitivity coefficient b. Cess [1976] has also developed empirical relationships for the dependence of outgoing infrared radiation on surface temperature that give values of the constants very similar to those used here when cloudiness is the same at each latitude.

The form of F adopted here is one of the parameterizations of Gal-Chen and Schneider [1976]. F_0 , F_a , and F_i are dependent on the local meridional temperature gradient. The empirical coefficients of proportionality at each latitude are designed to fit the current mean annual observations of F and $\partial T/\partial y$. Following the work of Stone [1974], the coefficients are taken to be proportional to the local temperature gradient. Thus F is proportional to the square of the local meridional temperature gradient. To incorporate the seasonal variation in the zonally equivalent mixed layer depth, R at each latitude is specified as a sinusoidal function with a period of 1 year. This is done in lieu of computing values from physical or dynamical considerations in order to see the importance of a time-varying R on the seasonal simulation. To add a mixed layer prognostic scheme would add an internal degree of freedom that would confuse interpretation of model results; however, this would be a useful next step for future simulations. Thus

$$R(\phi, t) = \overline{R}(\phi) + R^*(\phi) \sin \left[\omega(t - t_0)\right] \qquad (14a)$$

$$\partial R/\partial t = \omega R^* \cos \left[\omega(t-t_0)\right]$$
 (14b)

 \overline{R} is the mean annual thermal inertia of the top layer, and R^* is the annual cycle amplitude. The phase constant t_0 used is -3.2×10^6 s for the northern hemisphere and 1.3×10^7 s for the southern hemisphere. For t = 0 at December 1 and the frequency $\omega = 1.99 \times 10^{-7} \text{ s}^{-1}$ these values of t_0 place the minimum in R approximately 1 month after the summer solstice. This is in rough agreement with observations of the depth of the mixed layer [*Turner and Kraus*, 1967]. $R_t(\phi)$ is set equal to $2\overline{R}(\phi)$, or $\overline{R}_0(\phi) = \overline{R}(\phi)$. The determination of \overline{R} and R^* is addressed in the next section.

Given these parameterizations, (7) and (9) can be formulated as finite difference equations and are treated as by Schneider and Gal-Chen [1973]. The solution of (8) and (10) is by straightforward explicit Euler forward time differencing, with the predicted future value of T_0 depending on the present values of T_0 and T. The meridional grid spacing is 10° of latitude from pole to pole; the number of time steps used varies from 96 to 144 per year (for stability in the numerical scheme), depending on the values of R^* that are chosen. Model runs require 50-100 years of model time to reach 'equilibrium' after perturbations such as variations in the solar constant. This result will carry forward to any seasonal model-including computer costly GCM's-using realistic R and computing surface temperatures. (It should be possible, however, to save computational time by developing procedures which initialize a seasonal model close to an estimated equilibrium state. Of course, a model requires less simulation time the closer it starts to its equilibrium.) Convergence to equilibrium is accepted when the global annual storage of energy $(RT + R_0T_0)$ changes

TABLE 2. Observed Zonal Values of Net Radiation \overline{Q}_n and the Derived Northward Energy Transports \overline{F} , \overline{F}_a , \overline{F}_i , and \overline{F}_o

Latitude	Q_n , W m ⁻²	<i>F</i> , ×10 ⁷ W m ^{−1}	<i>F_a</i> , ×10 ⁷ W m ^{−1}		<i>F</i> ₀ ×10' W m⁻¹
85°N	-103.2	5.88	5.45	1.38	-0.95
75°N	-93.6	9.27	8.34	1.04	-0.11
65°N	-72.1	13.08	10.73	2.01	0.34
55°N	-46.6	15.32	8.81	3.97	2.54
45°N	-20.9	15.75	5.79	4.82	5.14
35°N	0.7	14.63	3.22	4.68	6.73
25°N	18.2	12.24	3.89	1.60	6.75
15°N	45.5	8.02	5.30	-3.49	6.21
5°N	58.9	2.17	3.41	-2.86	1.62
5°S	56.1	-4.08	-6.10	2.93	-0.91
15°S	40.7	-9.56	-7.17	1.63	-4.02
25°S	22.0	-13.75	-4.63	-1.90	-7.22
35°S	0.4	-16.54	-3.22	-4.68	-8.64
45°S	-27.3	-17.61	-5.66	-4.72	-7.23
55°S	-57.4	-16.65	-7.40	-3.33	-5.92
65°S	-85.6	-13.17	-8.51	-1.60	-3.06
75°S	-89.5	-8.48	-7.55	-0.93	0.
85°S	-87.7	-5.02	-4.02	-1.00	0.

				R*				
Latitude	f	(16))	(Model)	R 1	R2	R3	<i>R</i> 4	R5
80°N	0.81	15	14	0.	0.	0.	0.	0.
70°N	0.47	23	44	0.	0.	0.	0.	0.
60°N	0.38	23	42	0.	4.2	1.3	12.6	21.0
50°N	0.42	24	53	0.	5.3	2.7	15.9	26.5
40°N	0.54	29	58	0.	5.8	4.1	17.4	29.0
30° N	0.59	32	57	0.	5.7	4.6	17.1	28.5
20°N	0.68	41	66	0.	6.6	2.6	19.8	33.0
10°N	0.76	53	117	0.	0.	2.3	0.	0.
0°	0.77	56	128	0.	0.	1.3	0.	0.
10°S	0.78	57	153	0.	0.	1.5	0.	0.
20°S	0.76	52	90	0.	9.0	3.6	27.0	45.0
30°S	0.81	61	74	0.	7.4	7.4	22.2	37.0
40°S	0.95	126	245	0.	24.5	100.5	73.5	122.5
50°S	0.98	211	475	0.	47.5	132.1	142.5	237.5
60°S	1.00	533	477	0.	47.7	262.5	143.1	238.5
70°S	0.59	188	18	0.	0.	0.	0.	0.
80°S	0.05	15	18	0.	0.	0.	0.	0.

TABLE 3. Zonal Values of the Fraction of Ocean f, \overline{R} From (16), Model-Determined \overline{R} , and the Five Sets of R^*

All values except for f are $\times 10^6$ J m⁻² °K⁻¹. In these units, $\overline{R} = 100$ is equivalent to a column of water 24 m deep.

by less than 4.2×10^{-5} W m⁻². Further discussion of the approach to equilibrium is given in section 5.

The collection and processing of data for model initialization are described by Thompson [1977]. Temperature data were taken from the compilations of Schutz and Gates [1971, 1972, 1973, 1974]. The observed values of planetary albedo are those given by Ellis and Vonder Haar [1976], including our own assumed estimates where data were not available owing to the small amount of reflected radiation during the polar winter. Extraterrestrial insolation $Q(\phi, t)$ is from A. L. Berger (personal communication, 1976). The midmonth values of insolation and albedo were interpolated using a second-order accurate routine to the time grids used by the model. Mean annual values of observed surface air temperature \overline{T} , albedo $\overline{\alpha}$, and insolation \overline{Q} are given in Table 1. The observed mean annual total northward energy transport \overline{F} is derived from the annual net radiation at the top of the atmosphere \overline{Q}_n , as taken from Ellis and Vonder Haar. The procedure for estimating \overline{F}_a , $\overline{F}_{l}, \overline{F}_{0}$, and the empirical diffusion coefficients from the data of Oort [1971] and Vonder Haar and Oort [1973] is also given by Thompson [1977]. Values of $\overline{Q}_n, \overline{F}, \overline{F}_a, \overline{F}_l$, and \overline{F}_0 are given in Table 2.

The mean annual values of the zonal equivalent (or 'combined mixed layer,' as on Figure 1) thermal inertia $\overline{R}(\phi)$ must be derived from knowledge of the mixed layer depth and heat capacity of the ocean and from the heat capacity of the land and atmosphere. The thermal inertias of the atmosphere and land are taken as $R_a = 10.3 \times 10^6$ Jm⁻² °K⁻¹ and $R_l = 4.3 \times$ 10^6 Jm⁻² °K⁻¹, respectively, or approximately those of 2.5-m and 1.0-m depths of water. The mean annual thermal inertia of the ocean, \overline{R}_w , is determined from mixed layer depth estimates provided by J. Miller (personal communication, 1976) by the formula

$$\overline{R}_{w}(\phi) = \overline{Z}_{w}(\phi)\rho_{w}c_{w} \tag{15}$$

where \overline{Z}_{w} is the mixed layer depth, ρ_{w} is the density of water, and c_{w} is the specific heat. $\overline{Z}_{w}(\phi)$ can be inferred from Table 3, (15), and (16). At latitudes of 70° and poleward, \overline{R}_{w} is set equal to the thermal inertia of land, as a crude approximation allowing for sea ice coverage. As shown in Appendix A, it is difficult to assign a physical meaning to the combined equivalent zonal thermal inertia of the land, sea, and atmosphere. The resulting value of \overline{R} does not clearly pertain to any particular geographic component; therefore T does not represent the temperature of land, sea, or atmosphere separately. However, an area-weighted harmonic mean is one method of approximating \overline{R} and will first be used here (see Appendix A). Thus

$$\overline{R}(\phi) = \left[\frac{f}{\overline{R}_w + R_a} + \frac{1-f}{R_l + R_a}\right]^{-1}$$
(16)

where f is the fraction of the zone covered by ocean.

Considering the uncertainty in the values of the mixed layer depth (probably a factor of 2) and the restrictive assumptions used in deriving (16), it seems justifiable to allow the model to determine $\overline{R}(\phi)$, using calculated values from (16) only as a check on the model's results. Since the annual amplitude of the T curve at any given latitute is related inversely to \overline{R} for the latitude, the model can be run and \overline{R} adjusted until the observed and computed annual temperature amplitudes agree to within a close tolerance. Using the seasonally imposed (IS) albedos and the present solar constant scaling, a final set of $\overline{R}(\phi)$ is derived. When used in a seasonal simulation, these give an area-weighted rms error of 0.73°K between model and actual annual temperature cycle amplitudes. This final set of $\overline{R}(\phi)$ is used in all subsequent calculations and is given in Table 3 along with the $\overline{R}(\phi)$ from (16) and (15). As can be seen, the \overline{R} values obtained from optimizing the model's simulation of annual temperature cycle amplitude are generally a factor of 2 larger than the estimates based on (16), except south of 50°S. This probably occurs because the harmonic mean weights the R_i values too heavily (see Appendix A). In reality, the heat exchange by atmospheric transport between the land and ocean within a zone makes the thermal response of the zone more like that of the ocean than (16) would indicate. The hybridized, unphysical nature of \overline{R} is one of the principal limitations of a purely zonal model for seasonal simulation.

The appropriate values of the time-varying component of the thermal inertia R^* are even more difficult to ascertain.

				$\Delta \overline{T}$, °K		
Model	Albedo	Solar Constant	Thermal Inertia	Northern Hemisphere	Southern Hemisphere	Global
Annual Seasonal	MA	0%	R1	0.00 0.17	0.00 -0.19	0.00 -0.01
Annual Seasonal	MA	-1%	RI	-1.45 -1.28 (-1.45)	-1.46 -1.64 (-1.45)	-1.45 -1.46 (-1.45)
Annual Seasonal	MA	+1%	R 1	+1.45 +1.63 (+1.46)	+1.45 +1.26 (+1.45)	+1.45 +1.44 (+1.45)
Annual Seasonal	ATF	0%	R 1	0.00 +1.59	0.00 +1.85	0.00 +1.72
Annual Seasonal	ATF	-1%	R 1	-2.53 -0.86 (-2.45)	-2.52 -0.71 (-2.56)	-2.52 -0.78 (-2.50)
Annual Seasonal Initial annual mean tem-	ATF	+1%	RI	+2.58 +3.78 (+2.28)	+2.56 +4.09 (+2.24)	+2.57 +3.93 (+2.21)
perature				288.16	286.75	287.45

TABLE 4. A Comparison of the Mean Annual Surface Air Temperature Change From Initial Conditions Produced by the Annual and Seasonal Models

The numbers in parentheses are corrected for the inconsistency between initial conditions and equilibrium conditions for the case of a 0% change in the solar constant.

Hence we decide to bracket all likely values of R^* by forming five sets of values corresponding to $R^* = 0$ (R1), $R^* = 0.10\overline{R}$ (R2), observation-derived (but limited by uncertainties) values (R3), $R^* = 0.30\overline{R}$ (R4), and $R^* = 0.50\overline{R}$ (R5). For numerical stability (i.e., to reach an eventual equilibrium) the model needs 96 time steps/yr for R1, 120 steps/yr for R2 and R3, and 144 steps/yr for R4 and R5. R^* is set equal to zero near the poles to avoid the problem of sea ice variations changing the phase of the thermal inertia curve and also is set equal to zero in the tropics (except for set 3) because seasonal variations in temperature are small there. R3 is determined from the seasonal mixed layer depth data provided to us by J. Miller, using (15) and (16) to estimate seasonal R values. One-half the

TABLE 5. Differences Between Observed (Initial) and Experiment (IS, 0%, R1) Mean Annual Surface Temperatures, $\Delta \overline{T}$, Model and Observed Temperature Amplitudes, T^* ; and Their Differences

Latitude	$\Delta \overline{T}$	T* (Model)	T* (Observed)	Differ- ence	Relative Difference
80°N	1.07	16.8	16.1	0,7	0.04
70°N	0.86	15.1	15.9	-0.8	-0.05
60°N	0.82	13.3	14.7	-1.4	-0.09
50°N	0.77	11.9	11.9	0.0	0.00
40°N	0.65	9.4	9.3	0.1	0.02
30°N	0.41	6.4	6.6	-0.2	-0.03
20°N	-0.09	3.2	3.3	-0.1	-0.03
10°N	-0.51	1.7	0.9	0.8	0.85
0°	-0.90	0.5	0.7	-0.2	-0.26
10°S	-0.47	1.7	1.2	0.5	0.49
20°S	-0.30	2.9	2.5	0.4	0.15
30°S	-0.28	3.4	3.7	-0.3	-0.08
40°S	-0.23	3.0	2.8	0.2	0.05
50°S	-0.17	2.1	2.0	0.1	0.07
60°S	-0.21	2.2	3.7	-1.5	-0.41
70°S	0.83	10.4	10.3	0.1	0.01
80°S	1.33	18.3	14.5	3.8	0.26

All values except relative differences are in degrees Kelvin. Northern hemisphere $\Delta \overline{T} = 0.21^{\circ}$ K, southern hemisphere $\Delta \overline{T} = -0.22^{\circ}$ K and global $\Delta \overline{T} = -0.01^{\circ}$ K.

difference between the winter and summer values is taken to be the nominal R^* for R3. The five sets of R^* , together with $f(\phi)$ from *Sverdrup et al.* [1942], are given in Table 3. We recognize the large degree of arbitrariness in choosing these numbers, but what we are after is the sensitivity of the simulation results to a switch from one set of R^* to another.

The procedure described by Schneider and Gal-Chen [1973] by which $c(\phi)$ in (13) is obtained establishes consistency only for the case when $\partial R/\partial t = 0$ (i.e., when R is constant in time, the left-hand side of (1) is zero when averaged over a year). By 'consistency' in this context we mean no difference between the global annual temperature after running the model to equilibrium and the global annual temperature of the initial condition when using a solar constant scaling of 1.00. Because of the energy-conserving properties of (7)-(10), the global consistency is maintained when time-varying R is used. However, the latitude distribution of \overline{T} may change as changing seasonal temperature cycles influence the meridional heat flux parameterization. The initial values of T_0 used in the simulations are set equal to the presently observed values of $\overline{T}(\phi)$.

3. SIMULATION EXPERIMENTS

Comparison of annual and seasonal models. The number of experiments we have performed is large enough to warrant using an abbreviated system of nomenclature. The experiments can be divided into categories according to the particular albedo specification (either MA, IS, or ATF), the solar constant change used (+1%, -1%, or 0%), or the specified seasonally effective thermal inertia amplitudes (*R*1 through *R*5, as given in Table 3).

The annual/seasonal experiments are performed to compare the annual mean sensitivity of the seasonal model to that of an annual model for changes in the solar constant and albedo specification. The annual model is simply the seasonal model with the insolation at each latitude held constant at its mean annual value. Table 4 gives the mean annual temperature



Fig. 2. Seasonal cycles of temperature for four selected latitudes as observed (dotted-dashed lines) and simulated (solid lines) by experiment (IS, 0%, R1).

change from initial conditions for the comparison experiments.

As can be seen from experiment (MA, 0%, R1), the annual model is completely 'tuned' (i.e., consistent) for the present solar constant, but the seasonal model produces a slight rise in the northern hemisphere temperature (0.17°K) with a corresponding decrease in the southern hemisphere. (The Budyko meridional heat transport parameterization, which will be described later, produces no mean annual surface temperature change for either hemisphere when the present solar constant and mean annual albedos are used.) After adding this inconsistency back, the results of which are shown in the parentheses in Table 4, we see that there is virtually no difference between the annual and seasonal models when albedo is constant in time.

When albedo-temperature feedback is used, the temperature sensitivity increases from approximately 1.5°K to 2.5°K for a 1% solar constant change, using the annual model. The seasonal model shows a 1.72°K global temperature rise for the present solar constant. This increase results from the interaction of the time-varying albedo and insolation. More energy is gained by the combination of low summer albedo and high summer insolation than is lost by the opposite combination in winter. The absorbed solar radiation, when integrated over a year, is greater when the albedo varies seasonally. Thus there is a residual warming effect. (The seasonal model using IS albedo does not show this effect because $c(\phi)$ is derived using insolation-weighted annual mean values of $\bar{\alpha}(\phi)$ which correctly specify the absorbed solar readiation over a year.) This residual warming effect is important for problems such as the relation between orbital element variations and glacial/interglacial transitions, as discussed by Schneider and Thompson [1979]. After subtracting the 1.72°K warming we find a -2.50° K global change for a 1% decrease in Q and $+2.21^{\circ}$ K change for a 1% increase in Q.

At this point we may note some interesting, although relatively minor, differences between the sensitivities of the annual and seasonal models. These differences are due to various nonlinear effects such as the effect of differing areas over which ATF is active for differing global mean temperatures, a noticeable $\alpha(\phi)$ spacial truncation error effect in the annual model which is smoothed out in the seasonal model, and the effect of the nonlinear heat transport in redistributing global temperatures differently in a seasonal model as opposed to an annual model. In any case, these minor model idiosyncrasies notwithstanding, the sensitivities of both annual and seasonal models to small solar constant changes are similar.

Comparison with seasonal observations. The difference between the equilibrium mean annual temperature for the seasonal model (IS, 0%, R1) and the initial \overline{T} (i.e., $\overline{T}_{initial}$ - $\overline{T}_{equilibrium}$) is denoted by $\Delta \overline{T}$ and is given in Table 5. $\Delta \overline{T}$ in this case is a measure of how well the model is tuned to observed mean annual conditions. The amplitudes of the observed and model-derived temperature cycles (which are not tuned as a function of time) are also shown in Table 5, along with the difference and relative difference between the two sets. (Relative difference is defined as the amplitude difference divided by the observed amplitude; this becomes rather meaningless for small amplitudes.) Note that the model simulates the temperature cycle amplitudes well while using entirely reasonable values of time-invariant \overline{R} . When mean annual albedos are used instead of IS values, the northern hemisphere temperature cycle amplitudes decrease by about 10%, with a somewhat smaller decrease in the southern hemisphere. Since the seasonal temperature cycle is dependent on both the albedo cycle and thermal inertia, the same cycle of temperature can be derived using different combinations of these two factors. This implies that thermal inertia and albedo must be verified by some method other than model results of temperature computations.

Figure 2 compares the observed and (IS, 0%, R1) computed cycles of temperature for four selected latitudes. The observations are from Oort and Rasmussen [1971] for the northern hemisphere. The southern hemisphere data are from Taljaard et al. [1969]. Note that while the model reproduces the temperature amplitudes well, the model temperatures lag the observed values by 1-2 months. This temperature phasing is more characteristic of the sea surface than of the land-sea system. It may be possible to adjust $\overline{R}(\phi)$ to achieve the proper temperature phasing but probably not without resorting to unrealistic values and drastically altering the temperature amplitudes. (Later on we will note that this phase error is reduced by using R5 time-varying thermal inertias.) Either temperature amplitude or phase may be adjusted by varing R, with amplitude tuning producing the more realistic results. In any case, tuning R^* to produce the best fit would invalidate the use of the seasonal cycle as an independent check on model performance. It is probable that much of the excessive temperature lag results from treating a latitude zone as a composite of land and sea, and thus a more realistic seasonal simulation may require a model that divides each zone into separate land and sea components.

Taylor [1976] has shown that a heat conduction approximation to subsurface energy storage results in better surface temperature phasing than a mixing-type assumption. Applying such an approximation to the land in a land-sea model [e.g., *Sellers*, 1973] should result in a better overall zonal phase agreement. (The land surface, of course, is not a mixed layer.) The correct specification of heat conduction in the upper ocean is not straightforward. In many areas, however, using a variable depth mixed layer similar to that used here should be more realistic than using a variable thermal conductivity [e.g., *Saltzman and Vernekar*, 1971] if one can accurately predict the mixed layer depth.

The observed [Oort, 1971] and (IS, 0%, R1) derived values of the northward energy transports across each latitude are



Fig. 3. Total northward atmospheric energy transport for the four midseason months as observed (dotted lines) and simulated (solid lines) by experiment (IS, 0%, R1).

given in Figures 3-5. The total atmospheric energy transports (latent heat, sensible energy, and potential energy) for the four midseason months are shown in Figure 3. The agreement with the observations is generally good, except for October. The observed atmospheric energy transport for October is more similar to that of January than that of July, while the com-



Fig. 4. Northward latent heat transport by the atmosphere for the four midseason months as observed (dotted lines) and simulated (solid lines) by experiment (IS, 0%, R1).



Fig. 5. Mean annual atmospheric energy transports as observed (dotted lines) and simulated (solid lines) by experiment (IS, 0%, R1). (a) Latent heat. (b) Sensible heat plus potential energy.

puted transport is similar to the computed transport for July. Thus the model tends to carry the northern hemisphere summer well into what should be autumn. This has to be associated with the model's lagged temperature phase. But after accounting for the phase lag the annual shifting of the general circulation is simulated reasonably well.

Here we may stop to analyze more closely the nature of the differences between the simulations and the observations. We must first recognize that the zonal atmospheric meridional energy transport can be divided into two major categories: that due to the standing and transient eddies of the middle and high latitudes and that deriving from the large-scale meridional circulation, primarily the Hadley circulation. Upon observing Figure 3 we should probably discount October because of the temperature lag problem mentioned previously. The other months appear to be relatively unaffected, however. January and April show comparable differences between observations and computed values with no particular latitudinal trend. Differences in the tropics are no larger than those elsewhere. Considering that Figure 3 shows the computed and observed January and April curves to be qualitatively very similar, we cannot easily tell whether these 'errors' are model-derived or observational difficulties. Sellers [1973] notes that the fine structure shown by the observations are irregularities (of approximately 10×10^{14} W magnitude) that could easily derive from the small data base used.

July presents a different problem. The error clearly increases toward the tropics with the largest observable difference occurring at 5°S. This might have been expected because the diffusive energy transport parameterization has little physical justification in the tropics and because July represents an 'extreme' month, that is, a climatic change from the mean annual conditions to which the eddy diffusion coefficients are tuned.

The observed and computed latent heat transports are given in Figure 4; the mean annual latent and sensible plus potential energy transports are shown in Figure 5. While the middleand high-latitude seasonal comparisons show some skill, there are large errors in the tropics. (Note that the plotting of the figures may be deceptive; sloping lines that are close together tend to mask large differences.) Again, this could have been expected, since the atmospheric transport of latent heat in the tropics is not a diffusionlike process but is largely due to the direct low-level circulation. The fact that the model does not simulate well the seasonal variation of the meridional tropical latent heat transport is a good example of the need to use independent seasonal observations to verify a parameter-



Fig. 6. The seasonal cycle of temperature at two latitudes as simulated by (MA, 0%, R1) (solid lines), which uses time-invariant thermal inertia and (MA, 0%, R5) (dashed lines), which uses thermal inertia coefficients which are time varying. Note that the time of the summer temperature maximum in both hemispheres is closer to the maximum in summer insolation when time-varying seasonal thermal inertia is used (a considerable reduction of the phase errors seen in the simulations on Figure 2).

ization based on constants derived from mean annual observations. In this case the parameterization is clearly deficient in the tropics. This is easy to understand. The local meridional gradient of the mean annual zonal temperature $\partial \overline{T} / \partial y$ is used to define the diffusion coefficients in the transport parameterization. Since latent heat transports are large in the tropics and $\partial \overline{T} / \partial y$ is quite small there, the tropical latent heat diffusion coefficients must be relatively large. The tropical $\partial T/\partial y$, which is small on a mean annual basis, does nonetheless vary considerably seasonally, since the location of the maximum in zonal surface temperature tends to follow the maximum in zonal insolation. Thus computed zonal latent heat transports, which depend on the local meridional temperature gradient, will also vary greatly seasonally, resulting in an erroneous seasonal simulation. This is the cause of the result shown in Figure 4.

Budyko heat transport. In order to test the sensitivity of the model's seasonal performance to the parameterization of the meridional heat transport a supplementary experiment is performed using Budyko's [1969] expression for div F:

div
$$F = \beta[T(\phi) - \langle T \rangle]$$

Here the angle brackets denote a global mean, and β is 3.74 W m⁻² °K⁻¹. Hence the meridional energy transport is proportional to the difference between the local (zonal) and global mean temperatures. The supplemental experiment produces an area-weighted rms error in the amplitudes of the annual temperature cycles of 2.09°K in comparison with 0.73°K for (IS, 0%, R1), using the diffusive heat transport formulation. This is still relatively small (the area-weighted rms amplitude of the seasonal cycle of temperature is about 9°C in the northern hemisphere) and could be correctable to within the uncertainty of the \overline{R} values. The computed temperatures continue to lag the observed values by 1-2 months. Thus the formulation of the meridional energy fluxes does not appear to be of overriding global importance when compared with $\overline{R}(\phi)$ in determining the seasonal model's temperature performance. Of course, the temperature in the tropical latitudes undergoes



Fig. 7. The seasonal variation of T, R, and T_0 at 50°N for the (MA, 0%, R5) experiment using $R_t = 1.6\bar{R}$, $R_t = 2.0\bar{R}$, and $R_t = 2.4\bar{R}$. R is the same for all three cases. The equilibrium seasonal cycle of T is virtually unaffected by R_t (and is plotted as a single line). The dotted lines show that T_0 does change appreciably, however.

little seasonal variation, and this is the region where the diffusive types of parameterizations would cause the largest errors.

4. SENSITIVITY EXPERIMENTS

Seasonally varying thermal inertia influences on the temperature cycle. Since the thermal inertia acts as a scaling factor for the rate of temperature change, we might expect some modification or distortion of the annual surface temperature cycle when R is allowed to vary seasonally instead of being held constant in time. Figure 6 shows the annual cycles of Tfor 60°N and 60°S for the (MA, 0%, R1) and (MA, 0%, R5) experiments. We are comparing cases of constant R and maximally time varying R. It should be noted that the albedo is fixed at its mean annual value in order to examine separately the influence of seasonally varying R. Allowing R to vary so that its minimum value occurs in midsummer results in a more prominent peak in summer temperatures than is the case when constant R values are employed. The lower summer R5 values allow the temperature to rise more rapidly going into summer and cool more quickly going in autumn than is the case with

TABLE 6. Mean Annual Surface Temperatures \overline{T} , Bottom Layer Temperatures \overline{T}_{0} , and Their Differences for Model Experiment (IS, 0%, R3)

Latitude	\overline{T}	$\overline{T_{o}^{*}}$	$\overline{T} - \overline{T}_{0}^{*}$
80°N	258.28		
70°N	264.37		
60°N	273.03	267.16	5.87
50°N	278.78	273.51	5.27
40°N	286.46	282.38	4.08
30°N	293.41	290.77	2.64
20°N	297.89	296.59	1.30
10°N	298.66	297.92	0.74
0°	298.87	298.76	0.11
10°S	298.41	298.13	0.28
20°S	296.11	295.45	0.66
30°S	292.14	291.22	0.92
40°S	286.17	284.59	1.58
50°S	279.10	277.45	1.65
60°S	271.78	270.10	1.68
70°S	255.72		
80°S	235.10		

*Blank spaces in column indicate no seasonal thermal inertia variation.

	Solar Constant	Thermal Inertia Amplitude							
Albedo	Change	R 1	R2	R 3	<i>R</i> 4	R 5			
		Temperature Difference From Initial Conditions							
IS	0%	-0.ûl		-0.02		-0.03			
		(0.4541)		(0.4541)		(0.4522)			
IS	-1%	-1.46		-1.47		-1.48			
		(0.4512)		(0.4513)		(0.4493)			
IS	+1%	+1.44		+1.43		+1.42			
		(0.4569)		(0.4570)		(0.4550)			
ATF	0%	+1.72		+1.77		+1.78			
		(0.4105)		(0.4105)		(0.4079)			
ATF	-1%	-0.78		-0.68		-0.63			
		(0.4185)		(0.4179)		(0.4151)			
ATF	+1%	+3.93		+3.95		+3.95			
		(0.4053)		(0.4055)		(0.4030)			
MA	0%	-Ò.01	-0,01	-0.02	-0.02	-0.03			
		(0.4557)	(0.4556)	(0.4558)	(0.4553)	(0.4543)			
MA	-1%	-1.46	-1.46	-1.47	-1.47	-1.48			
		(0.4528)	(0.4527)	(0.4529)	(0.4524)	(0.4515)			
MA	+1%	+1.44	+1.44	+1.43	+1.44	+1.43			
		(0.4586)	(0.4585)	(0.4587)	(0.4582)	(0.4572)			
	Temperature Difference From Control Runs								
IS	-1%	-1.45		-1.45		-1.45			
	+1%	+1.45		+1.45		+1.45			
ATF	-1%	-2.50		-2.46		-2.41			
	+1%	+2.21		+2.18		+2.17			
MA	-1%	-1.45	-1.45	-1.45	-1.45	-1.45			
	+1%	+1.45	+1.45	+1.45	+1.46	+1.46			

 TABLE 7. A Comparison of the Global Mean Annual Temperature Change (°K) From Initial Conditions and From Control Runs

The global mean annual meridional temperature gradient in °K/100 km is in parentheses.

constant thermal inertia. We also see that the (MA, 0%, R5) curves are noticeably shifted so that the maximum temperatures occur more toward the time of maximum insolation and in closer phase agreement with observations (see Figure 2). The time of minimum temperatures is altered only slightly.

Figure 7 shows the seasonal variation of T, T_0 , and R at 50°N for the (MA, 0%, R5) experiment. Notice that T_0 is constant during the half year when R is increasing; T_0 varies only when the top layer thins and leaves mass behind in the bottom layer. The seasonal variation of T_0 is $\sim 3^{\circ}$ K when $R_t =$ $2\overline{R}$. As one would expect, the seasonal variation in T_0 increases (decreases) for $R_t = 1.6\overline{R}$ ($R_t = 2.4\overline{R}$). (When R_t is varied from $2\overline{R}$, the quantity which is changed is R_0 , not \overline{R} .) The fact that T_0 is below 0°C at times or that T_0 is sometimes greater than T (implying buoyancy instability) should not be dismissed as indicative of an unphysical parameterization. T_0 is not meant to be a realistic oceanic thermocline temperature; it is a composite land-sea subsurface temperature which is calculated to maintain zonal energy balance. A land-sea model is necessary to compute true thermocline temperatures. We also note that the seasonal cycle of T is slightly out of phase with the imposed cycle of R. In a model which explicitly computes both the temperature and depth of the mixed layer, these quantities should be closely in phase. A next step would be to compute R(t), not impose it.

Combination of albedo, thermal inertia, and solar constant specifications. In order to examine the interaction between various specifications for insolation, albedo, and thermal inertia a group of experiments is performed using various combinations of the specifications for these three elements. A total of 45 different experiments could have been done using the three possibilities for solar constant change and albedo and five sets of R^* . Of these, 33 runs were made.

Table 6 shows the mean annual equilibrium temperatures \overline{T} and \overline{T}_0 and their differences for model experiment (IS, 0%, R5). The \overline{T}_0 values are always less than \overline{T} , the differences increasing poleward. A similar result holds for all cases of time-varying R.

Table 7 gives the global mean annual surface temperature change from initial conditions and the global mean_annual meridional surface temperature gradient for some of the various experiments. The 0% solar constant change experiments can be considered comparison control runs for those experiments in which the change is $\pm 1\%$. The section at the bottom of Table 7 shows the global mean annual temperature difference from the control runs.

For the (IS, 0%) and (MA, 0%) experiments the temperature changes are zero (i.e., between -0.01° K and -0.03° K). This is to be expected, since the global radiation balance remains unaffected. The (ATF, 0%) experiments again show the previously mentioned rise in temperature when the seasonally varying albedo interacts with the seasonally varying insolation. Note that the use of R3 and R5 thermal inertia amplitudes increases the equilibrium temperature rise from 1.72° K to 1.77° K and 1.78° K, respectively, in the 0% cases. Here the summer temperature rises faster because of the low summer R values. This warming results in a decreased annual mean albedo and a concomitant rise in annual temperatures.

For an increase in Q the MA and IS experiments show the expected increase in meridional temperature gradient, due to a larger warming in the tropics than in the higher latitudes. This is reversed for the ATF experiments, with the meridional temperature gradients being negatively correlated with the solar constant changes [see Gal-Chen and Schneider, 1976].

Now we ask the question, Does the seasonal variation of R affect the model's response to solar constant changes? To



Fig. 8. The global rate of heat storage as given by *Ellis et al.* [1978] and simulated by the model, using various assumptions (on the figure). All curves are plotted using 3-month running means. Note that all curves have a similar phase except the model run for 11,000 YBP.

answer this, we compare the solar constant change runs with the control runs in the lower section of Table 7. A quick look shows that there is virtually no variation in the model's global response to solar constant changes of $\pm 1\%$ for any of the five R^* specifications. A closer look at the temperature response of each zone (not shown) also reveals that different sets of R^* make a negligible difference in the zonal sensitivity to solar constant changes. The particular specification of $\overline{R_t}$ could be expected to alter the amplitude of the seasonal cycle of T_0 and possibly to modify the equilibrium distribution of $\overline{T}(\phi)$ through changes in the meridional energy transport. Experiments using $\overline{R_t} = 1.6\overline{R}$ and $\overline{R_t} = 2.4\overline{R}$ show virtually no differences in $\overline{T}(\phi)$, however.

A traditional 'ice catastrophe' experiment (see, for example, North [1975b]) was performed using (ATF, R1). For this particular specification the seasonal model produces an ice-covered earth for a solar constant decrease of 3.45%. This response is very similar to that of the corresponding annual model.

The global rate of heat storage. In the model the global rate of heat storage is defined as the time rate of change of $R_0T_0 + RT$ averaged over all latitudes. Figure 8 shows equilibrium seasonal plots of this quantity for various model assumptions. Also given is the estimated global heat storage rate as derived by *Ellis et al.* [1978] from observations. The curves are plotted using 3-month running means. Model results using IS albedo and R1, R3, and R5 thermal inertias are all very similar and agree well with the observations. Use of mean annual albedos (not shown) gives results similar to those using IS albedos. Even reversing the latitude distribution of $\overline{R}(\phi)$ so that each latitude is assigned the $\overline{R}(\phi)$ value from the opposite hemisphere does not greatly alter the basic cycle.

Ellis et al. [1978] find the global rate of heat storage to be in close agreement with the global net radiation flux. As they explain, the global net radiation seasonal cycle is due largely to the varying earth-sun distance throughout the year. This variation, caused by the eccentricity of earth's orbit, results in about a 7.8 W m⁻² amplitude of variation in global absorbed radiation. Note that this corresponds well with the variation shown in Figure 8. To confirm that the seasonal cycle in global rate of heat storage is mostly orbitally induced; we have performed an experiment using the insolation of 11,000 years before present (YBP). At that time, perihelion occurred in July; presently, perihelion occurs in January. The resulting curve given in Figure 8 shows the phase of heat storage rate is shifted by about 6 months. Thus the basic curve of global heat storage rate appears to be orbitally induced with modulations caused by heterogeneities in earth's climate system.

5. TIME-DEPENDENT APPROACH TO STEADY STATE: IMPLICATIONS FOR ESTIMATING CLIMATIC IMPACT OF CO₂ INCREASE

Earlier we mentioned that it took some 50-100 years to reach a 'steady state' climate. In this section we refine that estimate quantitatively under a number of different assumptions of thermal capacities (R and R_0) and mixing coupling between upper and lower layers. Figure 9 shows the results of perturbing the model with a step function 2% solar constant increase after equilibrium has been established for the present solar constant. All runs in this section use imposed seasonal albedo. For R1 (no time variation in R) there is no exchange of energy between the upper and lower layers. Consequently, the approach to equilibrium will have a time scale inversely proportional to R. The 'e-folding' time is about 3 years for the R1 case, as can be seen by the curve labeled 'A' on Figure 9. Regardless of the thermal capacity of the deep layer, $R_0(R_0 =$ $R_t - \overline{R}$), the time-invariant R isolates the two layers. However, if time variation in R is allowed, the mixing between the layers causes the thermal capacity of the lower layer to be 'felt' by the upper layer. We saw this already on Figure 7. Curve B on Figure 9 shows how the coupling of the two heat capacities R(t) and R_0 through the moderate mixing implied by the R3 specification can delay the e-folding response of the surface layer temperature T from about 3 years to nearly 5 years. Deepening the lower layer to $R_t = 8\overline{R}$ further lengthens the efolding time of T to about 7.5 years (curve C on Figure 9). Note also that curves B and C for T_0 on Figure 9 show that the lower-layer temperature is still not yet fully at equilibrium, even after 500 years of simulation! Furthermore, the waiting time for T to proceed from 1/e of its final equilibrium value to a near equilibrium increases from decades (curve A) to centuries (curves C and D).

Finally, curve D is also for the case of $R_t = 8\overline{R}$, but R5 timevarying thermal inertia is used. Since the amplitude of R(t)increases with R5 as compared to R3, we would expect that this increased vertical exchange between the upper layer and



Fig. 9. The approach to equilibrium of upper- and lower-layer temperature after a +2% solar constant step function perturbation. Each case has reached equilibrium for the present solar constant at the moment of the perturbation.



Fig. 10. Time-dependent global annual surface (upper layer) temperature response to an exponential solar constant increase for varying model assumptions (on the figure). The solar constant increase is 1.38% at year 50, implying an equilibrium global warming of 2°K for this model.

the large thermal capacity of the deep layer would further hold back the approach of the upper layer towards equilibrium. Indeed, as Figure 9 shows for curve D, the *e*-folding time for T has increased to about 24 years.

The major potential consequence of interest from these experiments is that the delay in surface temperature response to an external forcing (e.g., a CO₂ increase) could be anywhere from a few years to a few decades or even greater, depending upon the relative thermal capacities and vertical coupling between upper and lower layers. Thus the absence of an expected temperature signal from, say, an observed CO₂ increase could easily be misinterpreted as a defect in the computed estimate of surface temperature response to the CO₂ increase, whereas in reality the computed estimate (assuming it was an equilibrium calculation like *Manabe and Wetherald*'s [1975]) might yet be validated decades later, as the climatic system approached equilibrium. Great care will be needed to model properly the coupling between upper mixed layers and the deep oceans to determine if years, decades, or longer is the real delay time.

To model the CO₂ question more realistically, we conduct one final experiment where we simulate the effect of an exponential increase in CO₂ with an exponential increase in solar constant Q_s (see, for example, *Hunt and Wells* [1979]). For our model, where $\beta_0 (= Q_s \partial \overline{T} / \partial Q_s) \sim 145^\circ$ K, we require a solar constant increase of ~1.014 for a 2°K steady state T increase (roughly equivalent to a doubling of CO₂ in 50 years). Thus we perturb the solar constant scaling at each latitude in the model with

$$Q_s(\tau) = Q_x(\tau = 0) e^{b\tau}$$
 $b = 2.74 \times 10^{-4} \text{ yr}^{-1}$

where τ is in years, and plot the results, as shown on Figure 10, for five cases of thermal inertia, increasing from zero to $R_t = 2\overline{R}$, $4\overline{R}$, and $8\overline{R}$. In all cases the R3 time-varying R(t) is used.

Again, we can see that after only $\frac{1}{2}$ century, delays of a few years to a decade or so are possible for these assumptions and longer delay times could be obtained if stronger coupling between upper and lower layers were included. Since very strong coupling between upper and lower layers is observed in the high-latitude oceans in reality and weaker coupling is likely in lower latitudes, these few early experiments can merely serve to point out qualitatively the need for better quantitative latitudinal resolution of the coupling in more realistic future simulations.

6. CONCLUSIONS

A zonal energy balance climate model with an interactive lower layer has been developed and is used to simulate the seasonal cycle through the use of seasonally varying insolation. The zonal seasonally effective or upper layer thermal inertia is allowed to vary seasonally to account for some of the effects of the seasonal variation of the oceanic mixed layer depth.

Our results show that mid-latitude meridional energy transports are simulated moderately well by an eddy diffusion approximation based on annual data. However, the diffusion form fails to predict the seasonal cycle of atmospheric energy transports in the tropics. In this region the diffusive parameterization should be replaced in future simulations with a more physically appropriate relation based on the large-scale mean circulation.

The equilibrium sensitivity of annual and seasonal models to solar constant changes is comparable. The seasonal variation of thermal inertia also has little influence on model global equilibrium temperature sensitivity. However, experiments employing step function and exponential solar constant increases show the time-dependent response of global surface temperature to be very dependent on assumptions of seasonal thermal inertia variation and lower-layer thickness. This uncertain global surface temperature lag time (anywhere from 2 to 25 or more years behind the solar constant increases) implies that modeling the time-dependent response to CO_2 perturbations, for example, will require a model that accounts both for the thermal inertia of upper and lower layers as well as for the degree of mixing between these layers.

Reasonable seasonal temperature and global heat storage rate simulations have been performed using this low-resolution climatic model. Our results demonstrate that a seasonal model must employ reasonable thermal inertia values (and a reasonable albedo variation) to simulate correctly the annual amplitude of the temperature cycle. This conclusion will carry forward to any model in the hierarchy of climatic models (including GCM's) if it attempts to simulate a seasonal temperature cycle by calculating surface temperatures. The experience gained here with this highly parameterized seasonal energy balance model demonstrates a somewhat depressing economic prospect for use of seasonal models with realistic seasonally effective thermal inertia for climate sensitivity studies because they will take decades to reach equilibrium if the climatic state is perturbed (for example, by CO₂ doubling). For explicit dynamical models (for example, GCM's) this may impose a formidable computer time obstacle to the use of seasonal models for climatic change studies-unless shortcuts such as estimates of the final equilibrium state are used as initial conditions.

APPENDIX A: EQUIVALENT THERMAL INERTIA FOR A ZONE

In order to determine how to weight the thermal inertias of the land and ocean areas to arrive at an equivalent thermal inertia for an entire zone we first write two simplified heat equations for land and ocean separately:

$$R_l(dT_l/dt) = S \tag{A1a}$$

$$R_w(dT_s/dt) = S \tag{A1b}$$

Here R_i and R_w are the land and ocean thermal inertias, and T_i and T_s are the land and ocean surface air temperatures, respectively. S is a heating function, assumed to be the same for both land and ocean. Note that there is no coupling between land and ocean nor coupling with adjacent zones. The equation governing the entire system is

$$R(dT/dt) = S \tag{A2}$$

where T is defined by

$$T = fT_s + (1 - f)T_l$$

and f is the fraction of the zone covered by ocean. Substituting the definition of T into (A2), we find that

$$R[f(dT_s/dt) + (1 - f)(dT_t/dt)] = S$$
(A3)

By replacing the derivatives in (A3) with (A1) we have

$$R \left[\frac{fS}{R_{w}} + \frac{(1-f)S}{R_{t}} \right] = S$$
 (A4*a*)

ог

$$R = \left[\frac{f}{R_w} + \frac{1-f}{R_l}\right]^{-1}$$
(A4b)

which is an area-weighted harmonic mean. Equation (A4) is, of course, only an approximation, owing to the assumptions of no land/sea heat transfer made in its derivation. However, other assumptions seem no less arbitrary, particularly considering the uncertainty in the values of R_w .

If there is complete and instantaneous heat transfer between the ocean and land, then T_s and T_l will be equal. In this case the determination of R is a simple calorimetry problem. The thermal inertia of a zone is just the weighted arithmetic mean of R_w and R_l :

$$\boldsymbol{R} = f\boldsymbol{R}_{\boldsymbol{w}} + (1 - f)\boldsymbol{R}_{l} \tag{A5}$$

As an example of the difference between (A4) and (A5), let f = 0.7, $R_l = 4.3 \times 10^6$ J m⁻² °K⁻¹, $R_w = 2.9 \times 10^6$ J m⁻² °K⁻¹, and $R_a = 1.0 \times 10^7$ J m⁻² °K⁻¹ (R_a is the thermal inertia of the atmosphere). After adding R_a to both R_w and R_l in (A4) and (A5) because the atmosphere is present over the land and ocean we get $R = 4.2 \times 10^7$ J m⁻² °K⁻¹ from (A4) and $R = 2.1 \times 10^6$ J m⁻² °K⁻¹ from (A5). Hence there is about a factor of 5 difference between the two approximations. R for the real earth lies somehwere between the extremes of (A4) and (A5), but these can be used as theoretical extremes in this work.

APPENDIX B: THE SPECIFICATION OF T_0

When thermal inertia is allowed to vary with time, a term must be included in the surface energy balance equation to account for heat transfer into or out of the layer from below. The top layer cannot gain or lose mass without consequences for energy balance. Mass changes imply energy changes (if $T > 0^{\circ}$ K) which must be accounted for elsewhere in the system. Mass that is gained by the top layer in the cooling season enters at the temperature T_0 .

Our original aim was to proceed 'one step at a time' to extend the annual model of *Gal-Chen and Schneider* [1976] to seasonal simulations. Adhering to this philosophy, we first decided to specify $T_0(\phi)$ initially and change it once each year by the same amount that $\overline{T}(\phi)$ had changed. There was no seasonal T_0 dependence. Of course, this method is simpler than the explicit lower-layer circulation used in this paper. With the proper initial $T_0(\phi)$ the experiments described here were performed using the seasonally invariant T_0 specification. The model sensitivities for such a specification are essentially the same as those shown here. It was only when another series of experiments not described here were performed that the deficiency of the T_0 specification became apparent.

Updating T_0 each year while maintaining no seasonal dependence is not an energy-conserving process. For experiments involving solar constant changes the inconsistency in annual energy balance is quite small. However, when the seasonal cycle of insolation is altered by a 25% increase in amplitude without changing the mean annual value [see Schneider and Thompson, 1979], the time-varying thermal inertia specifications with seasonally invariant T_0 result in an anomalous global warming. For example, when mean annual albedos are specified and a +25% insolation cycle amplitude change is imposed, there should be no global equilibrium temperature change for any R specification. This is because there is no change in absorbed radiation and, necessarily, there can be no change in outgoing infrared when the model is in energy balance. In fact, experiments using R3 and R5 thermal inertias show a 0.73°K and 1.07°K global temperature increase, clearly violating energy conservation.

For this reason we have used the two-layer system described here. Although the lower layer represents an increase in resolution from the 'classical' single-level energy balance models, it is necessary for performing general sensitivity experiments in which R can vary with time.

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